Sem III (INFT)(CBCS) AM III Dec 17 2.3/11/17

QP CODE: 24395

(3 Hours) Marks 80

N.B.1) Question no 1 is compulsory.

2) Figures to the right indicate full marks.

3) Attempt any three from Q2 to Q6.

Q1 a) If any 14 integers from 1to 26 are chosen then show that at least one of them is a multiple of another. 05

b) Functions f and g are defined as follows : 05

$f: R \to R$, $g: R \to R$ $f(x) = 2x + 3$, $g(x) = 3x - 4$.

Find fog and gofog.

c) $L\left(\frac{d}{dt}\frac{\sin 3t}{t}\right)$. 05

d) Show that there does not exist an analytic function whose real part is $3x^2 - 2x^2y + y^2$. 05

Q2 a) Evaluate $\int_0^\infty e^{-t}\left(\frac{\cos 3t - \cos 2t}{t}\right) dt$ 06

b) Evaluate $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)(s^2+9)}\right\}$ 06

c) Find bilinear transformation which maps the points Z=1, i,-1 into points W = i , 0, -i . Hence find fixed pts of transformation and the image of $|z| < 1$. 08

Q3 a) If A, B ,C are of subsets of universal set U, then prove that 06

A X (B U C) = (A X B) U (A X C)

b) Let A={1,2,3,6 } , B ={1,2,3,6,7,14,21,42} and R be the relation `is divisible by'. 06

Draw Hasse Diagram for two sets. Show that are posets.

c) Find Laplace transform of following functions. 08

(i) $e^{-2t}\sqrt{1 - \sin t}$ (ii) $te^{-2t}H(t-1)$

Q4 a) In how many different ways can 4 ladies and 6 gentlemen be seated in a row, so no ladies sit together. **06**

b) Find analytic function whose real part is **06**

$$\frac{\sin 2x}{\cos h2y + \cos 2x}$$

c) Evaluate inverse Laplace Transform of following functions **08**

(i) $\frac{1}{(s-3)(s+4)^2}$ by convolution theorem (ii) $\log\left(1+\frac{a^2}{s^2}\right)$

Q5 a) Solve the following equation by using Laplace transform **06**

$$\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t, \textit{given that } y(0) = 1$$

b) Find p such that the function $\frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\frac{px}{y}$ is analytic. **06**

c) For $x, y \in Z, xRy$ *if and if only* $2x + 5y$ is divisible by 7 **08**

is R an equivalence relation ? Find equivalence relation.

Q6 a) Each coefficient of the equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die. Find the probability that the equation will have real roots. **06**

b) A certain test for particular cancer is known to be 95% accurate. A person submits to the test and result is positive. Suppose that a person comes from a population of the 1,00,000 where 2000 people suffer from disease . What can we conclude about the probability that person under test has particular cancer? **06**

c) i) If five points are taken in a square of side 2 units. Show that at least two of them are no more than $\sqrt{2}$ units apart. **04**

ii) How many friends must you have to guarantee that at least five of them have their birthday in same month. **04**
